SOFTWARE DEVELOPMENTS FOR GEODETIC POSITIONING WITH GPS USING TI 4100 CODE AND CARRIER MEASUREMENTS

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ABSTRACT. With phase measurements on the carrier signals of GPS satellites a highly accurate relative geodetic positioning is possible. A main problem concerns the determination of the phase ambiguities. A method which helps to solve this problem using simultaneous P-code and carrier measurements is presented. After a sufficient time of observation pseudoranges with a noise level of a few millimeters are obtained. Software developed at the University of Hannover for the simultaneous determination of absolute and relative positions using these ranges is described.

Rockville 1985

GPS RECEIVER CODE AND CARRIER MEASUREMENTS

A GPS receiver reconstructs either the code, the carrier or both signals of an incoming satellite signal. Measurements are made on these signals through the code and carrier tracking loops. The code tracking loop correlates the incoming code with a code replica generated by the receiver. If maximum correlation is reached the phase of the code replica is a measurement of the received code phase at a time event of the receiver clock. In the carrier tracking loop the relative phase between the reconstructed carrier signal and a reference signal generated by the receiver is measured:

$$\varphi_M = \varphi_C - \varphi_0 \qquad (1)$$

where

φ_M - is the measured relative phase,
φ_C - the received carrier phase and
φ_0 - the phase of the reference signal.

With

$$\varphi_0 = t_{C,R} \cdot f_0 \qquad (2)$$

the carrier phase can be computed by

$$\varphi_C = \varphi_M + t_{C,R} \cdot f_0 \qquad (3)$$

It is

$t_{C,R}$ - the receiver time at the time of the carrier reception and
f_0 - the nominal frequency of the reference signal.

If the receiver time and the reference signal are derived from the same oscillator there will be no error in the computed phase of the reference signal. Once the carrier signal is acquired and no loss of signal occurs, the phase change can be measured, including whole cycles.

Since the phase of a received signal equals the phase of the transmitted signal (Remondi 1984), the receiver measurements are observations of the signal phases at the satellite and the corresponding transmission times. The transmitted signals are derived from the satellite oscillator. Thus, the phase of a signal represents the satellite time. From a code phase the code transmission time can be computed by

$$t_{P,S} = \Phi_P / f_P \qquad (4)$$

Here
$\Phi_{P,S}$ - is the (p-) code phase (Φ_P $(t_S=0)=0$)
f_P - the nominal code frequency and
$t_{P,S}$ - the satellite time of code tranmission.
The carrier transmission time is given by

$$t_{C,S} = \Phi_C / f_C \qquad (5)$$

where
Φ_C - is the carrier phase since t =0 (Φ_C $(t_S=0)=0$) and
f_C - the nominal carrier frequency.

Thus, both the code and carrier phase measurements can be treated as observations of the satellite time at the corresponding time of phase transmission.

A first difference between the two phases is that the code phase can be measured without ambiguity but the carrier phase is ambiguous. There is an integer number of cycles unknown in the first observed carrier phase.

$$\Phi_C = \varphi_C + N_0 \qquad (6)$$

This is the ambiguity problem.

The second difference is the ionospheric propagation time delay. Let n be the refraction index for carrier phases, then the group refraction index for the code is

$$n_g = n + \frac{dn}{df} f \qquad (7)$$

So the difference in propagation times of code and carrier phases is

$$\Delta T_I = \int_{t_t}^{t_r} \Delta n \, dt \qquad (8)$$

with

$$\Delta n = n - n_g \qquad (9)$$

$$\Delta T = T_C - T_P$$

where
T_C, T_P are the propagation times.

Another difference is the noise level of the measurements. For example, the noise of TI 4100 p-code measurements is about 2 nsec (- 0.60 m), the noise of the carrier phases about 0.01 nsec (- 3 mm).

PHASE AMBIGUITY DETERMINATION FROM CODE AND CARRIER MEASUREMENTS

The following equations are derived from the above relations:

$$t_{S,I} = t_{C,S} + \Delta T_{C,I} = (\varphi_C + N_0) / f_C + \Delta T_{C,I} \qquad (10)$$

$$t_{S,I} = t_{P,I} + \Delta T_{P,I} = \Phi_P / f_P + \Delta T_{P,I}$$

and

$$N_0 / f_C = \Phi_P / f_P - \varphi_C / f_C + \Delta T_{P,I} - \Delta T_{C,I} \qquad (11)$$

It is
$t_{S,I}$ - the space vehicle time corrected for ionospheric propagation time delay,
$\Delta T_{C,I}$ - the ionospheric time delay for the carrier phase and
$\Delta T_{P,I}$ - the ionospheric time delay for the code phase.
The accumulation of equation (11) for all times of observation yields

$$N_0 / f_C = \Sigma(\Phi_P / f_P - \varphi_C / f_C + \Delta T_{P,I} - \Delta T_{C,I}) / m \qquad (12)$$

with
m - the number of observations.

To evaluate this equation the sum of ionospheric time delay has to be known. If one neglects higher order effects the ionospheric model

$$\Delta T_{C,I} = - A / f_C^2 \qquad (13)$$

where
A - is a factor varying with time and location,
results with (7) in

$$\Delta T_{P,I} = A / f_C^2 \qquad (14)$$

and

$$\Delta T_{P,I} - \Delta T_{C,I} = 2 A / f_C^2 = 2\Delta T_{P,I} \qquad (15)$$

With two-frequency code measurements the ionospheric delay can be computed from

$$\Delta T_{P,I,1} = - (t_{P,S,1} - t_{P,S,2}) / (1 - f_{C,1}^{2} / f_{C,2}^{2}) \tag{16}$$

(subscription 1,2 for L1, L2 respectively).

The standard deviation of the ambiguity computed from equation (12) with uncorrelated code and carrier measurements is

$$\sigma_{N/f} \approx 5 \sigma_{P} \tag{17}$$

where σ_P is the standard deviation of the code measurements.
With a standard deviation of about 2 nsec for TI 4100 code measurements this results in $\sigma_{N/f}$= 14.4 cycles. The accumulation of 1000 measurements is neccessary to get the ambiguity with a standard deviation of 0.5 cycles. The mean output rate of the TI 4100 is about 4 seconds. Thus, a continuous observation of one satellite over 1.2 hours is required to compute the ambiguity with the above accuracy. A higher output rate would be worthwhile for reducing observation times.

Figure 1 is a plot of the determined ambiguities for one satellite versus time at two simultaneously observed stations. The dotted curve is the ambiguity and the continuous line the standard deviation. It is obvious that there is a high correlation between the two stations. Similar behaviour can be observed with other satellites. There seems to be a systematic error in the model.

The above model does not account for phase delays and phase center differences between the different carrier and code signals. However, phase errors due to satellite hardware will affect all simultaneously observing stations in the same manner. These errors are independent of station locations and should vary slowly. For multiplex receivers like the TI 4100 phase delay errors are more or less the same for all observed satellites because there is only one hardware channel for all tracked signals. Nevertheless there are possible differences between different frequencies, so further analysis is required in this respect. Two different receivers of the same type may have similar errors, however, the change with time should be uncorrelated.

Another error in the described model is caused by the neglected ionospheric effect of higher orders. There is no complete agreement among many people concerned with this problem about the order of magnitude of these effects. With simultaneous code and carrier measurements the change in first order ionospheric refraction can be computed using the two-frequency model in two independent ways. But again the accuracies of the two determinations are separated by about two orders of magnitude. It can be seen from carrier measurements that the change in the refraction is very smooth. Figure 2 shows a second order polynomial fitting of the change in the ionospheric refraction determined from carrier as well as from code measurements on the two simultaneously observed stations (with a distance of 5 km). Significant coefficients were obtained from both approximations. The third plot shows the elevation angle of the satellite. The residual error sum of the two-frequency correction in the change of the ionospheric refraction was about 2 nsec here. The first order refraction amounted to 15 nsec at the beginning of the interval. The absolute error in one time correction can not be computed from these measurements. To estimate this quantity a model has to be adopted.

Through the algorithm about twice the mean residual error in the code refraction minus the error in the carrier refraction will be contained in the ambiguity. It seems that this can result in some cycles. However, if short baselines are observed the error should be the same on both sides.

With the computed ambiguities one gets the satellite transmission times with the noise level of the phase measurements (a few millimeters) and with a possible bias of a few cycles. From these times corresponding pseudoranges can be computed and used in an adjustment procedure similar to a model using pseudoranges obtained from code measurements.

THE GEONAP SOFTWARE

Software for geodetic positioning using the described model is being developed at the Univerity of Hannover. Some parts of the software depend on the TI 4100 data structure, others do not. The data flow chart gives a general view of this GEONAP (GEOdetic NAvstar Positioning) software. A brief description of the function of the modules follows.

TIDECO:
This program uses files with the raw data collected from a TI 4100 NAVSTAR Navigator. It decodes the binary data stream and separates different data types like the raw measurements, the NAVSTAR navigation message, TI 4100 user solutions and other information.

CYCLAM:
This software automatically edits the raw measurements, detects cycle slips in the carrier measurements and computes the ambiguities of carrier phases. One option for this program is to use one or both carrier frequencies. The cycle slip detection works in a way that the carrier phase is predicted for the next observation time and compared with the actual measurement. The prediction model is a polynomial computed from the previous measurements. The prediction accuracy is good enough for the short measurement intervals of the TI 4100. Thus, for this receiver the program works without knowledge of station coordinates and satellite orbits. For receivers with lower output rates a modified model may be neccessary. The output of CYCLAM is a file with edited measurements and another file with control information about the ambiguities.

IOCORR:
This module adds the computed ambiguities to the carrier phases and computes and corrects for the ionospheric refraction. The output rate for the corrected measurements is controlled by an input parameter, so a reduction of the number of measurements is possible. A further option is installed to choose the use of L1, L2 or two-frequency measurements. This allows an analysis of uncorrected ionospheric effects in station coordinates. The software is prepared to work with an adopted ionospheric model. This will probably be neccessary if only one-frequency measurements are available and longer baselines shall be determined.

MSSORT:
MSSORT uses the NAVSTAR navigation message files from all stations observed at the same time, checks the decoded messages for completeness, reduces redundant information and outputs a file with sorted messages.

The output file of MSSORT is used in this software to compute smoothed satellite orbits using least squares fit algorithms with polynomial models. The first reason for this is to represent the satellite coordinates without jumps as they occur in the navigation messages every time a new message set is transmitted. The second reason is to have the possibility of interpolating orbits if a navigation message is missing. A third reason is to have no interference with other software parts if different orbit information is available or different orbit modelling is desired.

SINCOR:
The SINCOR program uses the ionospheric corrected measurements, the smoothed orbits and an optional weather file as inputs. Corrections like satellite clock offsets, relativistic time effects, earth rotation effects and tropospheric refraction are made on the measurements or satellite coordinates respectively. A single station solution for every observation time and an accumulated least squares solution for station coordinates and a polynomial receiver clock model are produced. Orbit parameter partial derivatives, with respect to ranges, are determined and written to an output file together with the corrected measurements and satellite coordinates.

GEONAP:
This final software is a multiple station adjustment program. The output files of SINCOR are used to do a modified least squares adjustment of the satellite time observations. The parameter vector contains
- 3 coordinates / station,
- polynomial clock parameters for receiver clocks,
- tropospheric parameters,
- up to 6 keplerian orbit parameters / satellite
- polynomial clock parameters for each satellite clock and
- 1 ambiguity parameter / station / satellite.

There is no fixed length for this vector. Options are used to choose parameters to be estimated. The program is prepared for the introduction of additional parameters. A covariance model for receiver and satellite clock errors is used to describe integrated random noise oscillator errors. A priori variances or covariance matrixes for single parameters or parameter blocks are optional inputs. This allows the usage of prior adjustments and the fixing or constraining of parameters.

The detailed observation equations and covariance models for pseudoranges shall not be discussed in this paper. There are a lot of publications dealing with this subject.

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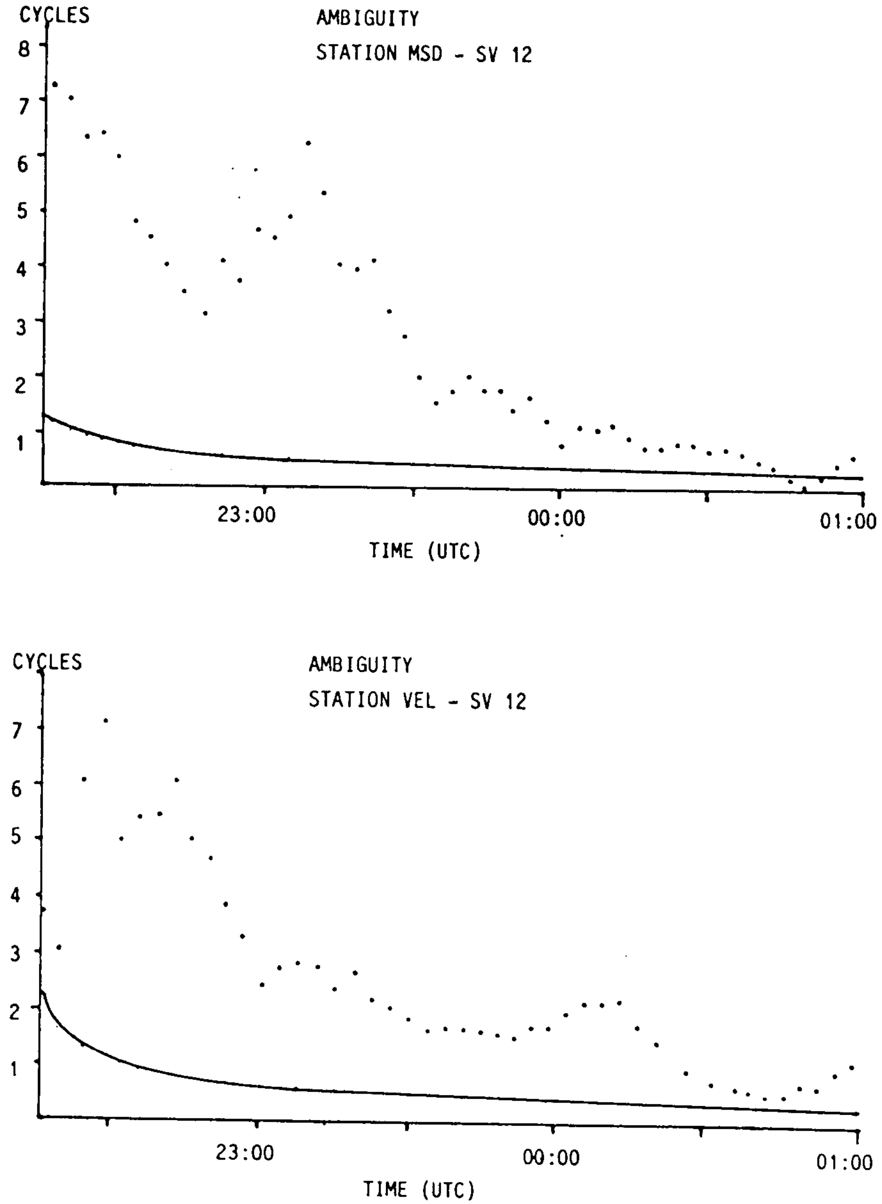

Figure 1: Ambiguities and Standard Deviation

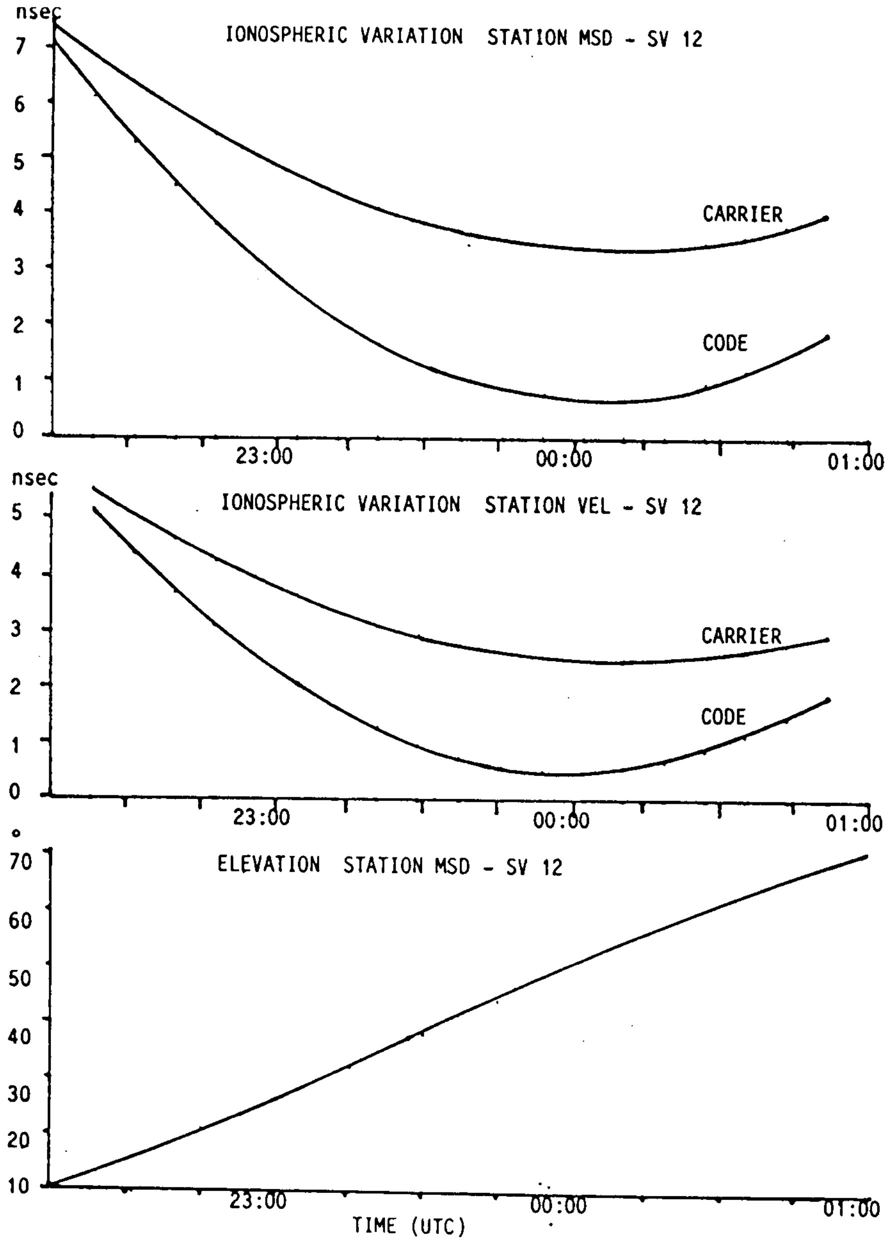

Figure 2: Ionospheric Variation in Code and Carrier Refraction

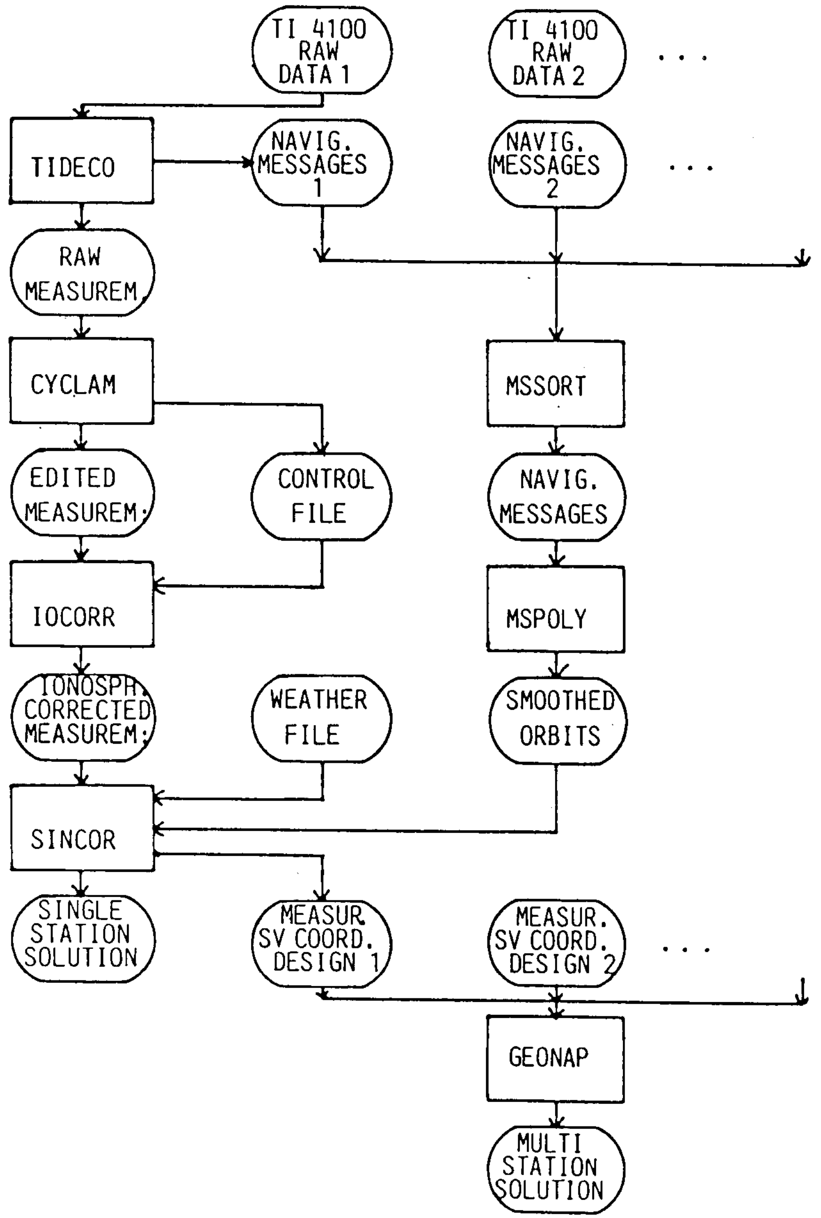

Figure 3: The GEONAP Data Flow Chart