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**Benefit of Rigorous Modeling of GPS  
in Combined AT/GPS/IMU–Bundle Block Adjustment**

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**Abstract**

The benefit of a rigorous GPS modeling in the combined bundle block adjustment has already been investigated some years ago. However, the closed GPS approach is only used operationally in the subsequent processing with the GEONAP–K package for GPS data and with the BINGO–F package for the combined adjustment. Recently, the BINGO–F package has been extended for the combined adjustment of additional IMU (Inertial Measurement Unit) data.

The rigorous GPS approach in a combined GPS/block adjustment uses the actual GPS constellation for the determination of projection center and does not rely on approximative shift and drift parameters, which are generally applied. The advantage is the geometrical constraint of the projection centers within the complete block or at least between individual strips under unfavorable GPS conditions. Changes in satellite constellation do not affect the combined adjustment. The geometrical information from GPS for neighboring strips or the complete block is maintained and strengthen the combined adjustment. The theory of the rigorous GPS modeling will be discussed.

For the integrated sensor orientation the correct modeling of all sensor is an essential task. The rigorous GPS approach in a combined bundle adjustment together with IMU and photogrammetric data will consequently also benefit. The European Organization for Experimental Photogrammetric Research (OEEPE) has conducted a multi–site test for the integrated use of AT (Aerial Triangulation), GPS and IMU data. Based on the test, analysis are presented, which focus on the effects of the GPS modeling in the combined bundle block adjustment with the GEONAP–K and BINGO–F software packages.

## 1 Introduction

The integration of the Global Position System (GPS) into photogrammetric projects is commonly applied. Besides GPS navigation and GPS ground control surveys, the major interest is the determination of the coordinates of the projection center as part of the photogrammetric exterior orientation. The combined GPS/block adjustment used for this task is a state-of-the-art technique and is used operationally in aerial triangulation.

A further reduction of costs is expected from the integration of Inertial Measurement Unit (IMU) data to determine the complete exterior orientation including the orientation angles of the camera during aerial triangulation. These new attempts make it necessary to analyze the currently used models of the integrated AT/GPS adjustment.

The current constellation of GPS of 29 satellites tends to neglect remaining problems in the general processing of kinematic GPS data. There are still GPS constellation changes during a flight from strip to strip. The so-called shift & drift approach is often applied in the combined GPS/block adjustment, which has the task to account for systematic GPS errors. Discontinuities in the determined GPS trajectory are caused by constellation changes, while time dependent changes originate from unreliable or false ambiguity resolution. The effects can only be approximated by the shift and drift parameters, while the strips are not too long and the magnitude and variations of the errors are not too high. There exists also a high correlation of the shift and drift parameter with other parameters of interest, which makes it impossible to estimate such parameters correctly.

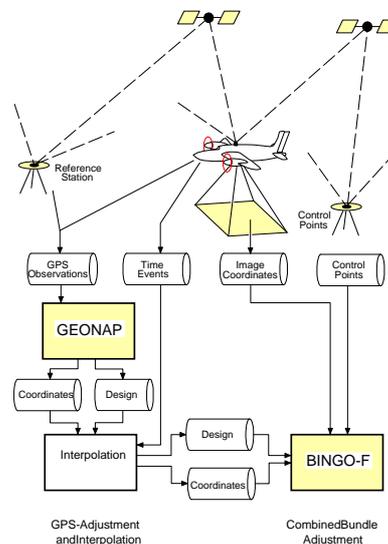


Figure 1: Data flow of GEONAP–K and BINGO–F (from Kruck et. al. 1996)

A mathematical model performs generally best, if a closed functional relationship is used or remaining approximation errors are small. If the approximation error may reach the magnitude of the actual error component, the individual error must be separated and adequately modeled for

highest accuracy requirements. The philosophy of separation error components is incorporated into the rigorous GPS modeling approach in the combined bundle adjustment with the GPS processing package GEONAP–K and the bundle block adjustment BINGO–F.

A rigorous GPS modeling is applied in the combined block adjustment to overcome the approximation of the shift & drift approach and the correlation with other parameters. In addition, the geometric strength of the GPS positions is maintained and the geometric information in the combined GPS/block adjustment is constrained from this fact. The approach is also termed CPAS (Combined Phase Ambiguity Solution) in the combined GPS/block adjustment with BINGO–F. The rigorous GPS model has been described by Kruck et. al. (1996), Jacobsen, Schmitz (1996) and Schmitz (1998). Empirical results are discussed in e.g. Okamoto (1998).

The European Organization for Experimental Photogrammetric Research (OEEPE) has conducted a multi–site test for the integrated use of AT, GPS and IMU data (Heipke et. al. 2000). The original idea of this paper was the description, application and discussion of the rigorous GPS model using GEONAP–K and BINGO–F using the test data. For this purpose, the data including the recorded raw data of all GPS receivers is required. Although, the description and investigation of all available techniques and methods is the goal of the OEEPE test, the necessary data was not accessible through the pilot center. The presented analysis uses the photogrammetric and GPS raw data of the IGI flight from the OEEPE test, which has been provided by IGI. Some analysis are presented from the complete photogrammetric data of the test, but the IMU data are not used. For detail on the OEEPE test, objectives, participants and configuration see Heipke et. al. (2000, 2001).

## **2 Systematic GPS Coordinate Errors**

Static GPS and realtime application of GPS can routinely achieve an accuracy at the few centimeter level, and, for certain applications even well below one centimeter (i.e. Wübbena, Lahr 2000). In contrast to static GPS measurement, no accumulation of measurements is possible in the determination of a kinematic trajectory. Therefore the processing of kinematic GPS station is still a challenging task. The accuracy of kinematic GPS for dynamic application depends on the distance to the reference station, the used observable and also on the processing strategy. In the following, always the highest accuracy requirements for the GPS processing is assumed.

The distance dependent errors are the ionosphere, troposphere and orbits. With increasing distances to the GPS reference station, the reliable ambiguity resolution becomes more difficult. Ambiguity resolution is the key issue to get an accuracy at the several centimeter level. The distance dependent errors can be modeled in the GPS processing package GEONAP–K.

Additional systematic GPS coordinate errors are generally caused in high dynamic kinematic applications by false ambiguity fixing, unresolved ambiguities and changes in the satellite constellation. The quality of the ambiguity resolution is steadily improving, but satellite constellation changes generally occur during a flight. Avoiding a loss–of–signal can be attempted during curve flights of the plane, but signal interruptions are often still present in the data. Automated data reduction in the GPS processing may introduce additional constellation changes not expected from the visibility of satellites at the kinematic station.

The magnitude of shift and drift effects in dynamic GPS applications depends on the actual geometric GPS conditions. The measure for this are the dilution of precision (DOP) values of GPS, which are generally given for geometry, called GDOP, or the position, called PDOP. Values of 3 or

less indicate very good conditions. Nowadays, the GPS satellite constellation is mostly favorable, so that the amplification by a poor DOP values is today mostly small. The effect must also be compared with the actual accuracy requirements of the photo flight or the intended accuracy for georeferencing.

Nevertheless, the GPS processing software must be capable to account for all possible error components. GEONAP-K allows a simultaneous multi-station, multi-frequency adjustment of the undifferenced GPS observable, which make the ambiguity resolution and the modeling GPS error components much more flexible. A closed simultaneous adjustment of several reference stations and several kinematic stations is possible, which is ideally suited applied with permanent reference station data. Combined adjustment of single and dual-frequency GPS data allows the ionospheric correction of e.g. a single frequency receiver in the photo flight airplane. Table 1 compares different scenarios of a local reference station, one remote reference station and a reference station network and some options to model systematic GPS errors.

<i>Kinematic GPS Processing</i>	<i>local reference station</i>	<i>remote reference station</i>	<i>reference station network</i>
ambiguity resolution	possible	difficult	possible
distance dependent errors:			
• ionosphere	ignore, eliminate	ignore, eliminate	model, eliminate
• troposphere	(model)	(model)	(model)
• orbit	(PE)	(model, PE)	(model, PE)
remaining systematic effects:			
• shift, drift errors	(approximate,) model	(approximate,) model	(approximate,) model
costs	high	low	low

Table 1: Photo flight configuration of GPS reference stations and comparison of different aspects of processing and costs

The use of a local reference station is favorable for the ambiguity resolution and therefore for the accuracy and simplicity of processing, but it is very cost intensive. The use of remote reference stations, which generally operate permanently, reduce the logistical and operational burden dramatically as well as the cost. However, ambiguity resolution and distance dependent errors increase and degrade the accuracy level. An additional improvement is gained from several reference stations, which can be processed as a reference station network. It is then possible to achieve ambiguity resolution over longer distances, while e.g. applying ionospheric modeling. Orbit improvement techniques can also be introduced in a network, without the delay of precise ephemeris (PE). Some GPS error components may be ignored, but may then introduce addition coordinate errors. The remaining systematic GPS effects cannot be approximated or modeled without any redundant observation and is therefore part of the combined GPS/block adjustment.

### **3 Modeling of Remaining Systematic GPS Coordinate Effects in the combined GPS/Block Adjustment**

It is a common procedure in the combined GPS/block adjustment, to reduce all efforts in the GPS processing and to approximate all systematic GPS errors as a lump sum, while applying shift and drift parameters. The method is often called shift & drift approach. This is the false strategy considering highest accuracy by separating and correctly modeling individual error components. To point out the major important aspects, the generally applied approximative shift & drift approach for correction of systematic GPS errors will be discussed in comparison to the rigorous GPS modeling approach.

All distance dependent GPS errors can best be modeled in the GPS processing, exceptionally with a sufficient number of reference stations and an adequate software package. Remaining systematic GPS effects due to the high dynamic photo flight and its presence in the GPS data require an adequate modeling, especially with respect to the combined adjustment of GPS and aerial triangulation.

The basic concept of the shift & drift approach is a linear regression of the systematic GPS effects and errors. The Systematic effects of the GPS coordinates (and often systematic error from atmosphere and orbits) are approximated by constant and time dependent coordinate corrections generally for every strip or simplified for the complete block. It is generally not accounted for effects due to satellite constellation changes in the combined adjustment nor in the GPS processing.

The best choice for the formulation of the combined GPS/block adjustment is the object space. The centered GPS coordinates correspond to the coordinates of the projection center. The coordinates of the external orientation from photogrammetric data can be used as redundant observation in the adjustment and vice versa. The formulation of the combined adjustment in the image space is also used, but has the major disadvantage, that the linear dependence of image coordinates (internal orientation) and projection center (external orientation) are used to express changes of external orientation by changes of the internal orientation in the image space. As a consequence, the separation from other parameters is difficult due to high correlation and is only possible, when it is applied for different time dependent parts of the data set.

When the shift and drift parameters are used strip wise, no geometric GPS relationship between strips exists anymore. Every strip or sub-block with an individual set of shift and drift parameters is completely independent from each other, because the introduced parameters destroy the geometric constraints from GPS. Even neighboring strips or repeated strips are completely independent concerning the GPS data, if individual shift and drift parameters are applied and the GPS position are translocated and scaled.

The systematic GPS errors can generally not be determined from a sub-set of data for a complete trajectory of a moving GPS receiver. Therefore uncertainties will remain, if no adequate modeling or configuration of the photo flight is used. Also the general accuracy requirements must always taken into account for the processing strategy. To be able to control the error behavior of the systematic GPS errors at least one, favorable some ground control points must be available.

### **4 Rigorous GPS Model for Combined GPS/Block Adjustment**

In the following, the rigorous GPS model for the combined GPS/block adjustment is described.

The redundant information of the coordinates of the projection center from photogrammetric data and GPS can be used in the combined adjustment. The general distance dependent errors of GPS have been correctly modeled in the GPS processing. Additionally, GPS position correction due to the remaining shift and drift effects are required. A simplified design matrix for a GPS adjustment model can very easily be computed from elevation and azimuth of all satellites used for the position estimation in the GPS processing. To estimate a position correction of the GPS trajectory, only the not reliably resolved ambiguity have to be known for every position. Range corrections for these satellites are introduced as unknown into the combined adjustment, which give with the design information a coordinate correction using strictly the functional model of the actual GPS constellation. Reliably resolved ambiguities of the GPS processing are unchanged and are still used for the GPS coordinate correction, but must not explicitly be known.

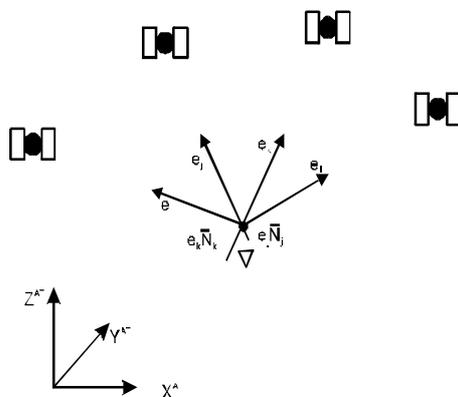


Figure 2: Principle of rigorous GPS modeling in combined GPS/block adjustment: projection center  
 • AT and ∇ GPS

The principle of the rigorous GPS model is displayed in Figure 2. The design information actually gives the unit vectors  $e$  in direction to the GPS satellites  $i$  to  $l$ . For the unresolved ambiguity term  $N$  of satellite  $j$  and  $k$ , a range correction is then estimated.

The coordinate corrections are computed using design information and estimating the ambiguity terms within the combined GPS/block adjustment. There exists a difference in the datum between GPS and the reference system of the photogrammetric object space. Therefore, a term for a datum transfer is required in addition to the remaining systematic GPS effects. The GPS positions are considered as observations in the combined block adjustment. The complete model for the rigorous GPS modeling in the combined GPS/block adjustment reads:

$$\mathbf{X}_p^{AT} = \mathbf{X}_A^{GPS} + \mathbf{dX}_D + (\mathbf{QA}^T \mathbf{P})_i \times \mathbf{N}_i + \mathbf{R}_i (\varphi\omega\kappa) \times \mathbf{dX}_A$$

The GPS coordinates  $\mathbf{X}_A^{GPS}$  of a position  $i$  are transferred to the coordinates of the exterior orientation applying  $\mathbf{X}_p^{AT}$  the eccentricity of the GPS antenna  $\mathbf{dX}_A$  with the rotation matrix of the camera  $\mathbf{R}(\varphi\omega\kappa)$ , the datum difference  $\mathbf{dX}_D$  and the position correction of the rigorous GPS model computed from the design  $(\mathbf{QA}^T \mathbf{P})$  and the unsolved ambiguity term vector  $\mathbf{N}$ . The GPS coordinate correction term actually accounts for range correction from the current satellites constellation.

Generally, the number of additional parameters for the correction of systematic GPS effects is smaller compared with a shift & drift approach, because not all signals are lost during every curve flight. Hence, only a minimum of required parameters has to be estimated in the adjustment.

The datum difference (datum transformation) can be described as translations only, or can be incorporated with a complete seven-parameter-transformation depending on the actual data set. The orientation angles  $\phi\omega\kappa$  are used from the exterior orientation or from an IMU data, to reduce the GPS positions given for the antenna phase center to the photogrammetric projection center. In modern systems also the crop angle is measured and can correctly be applied.

The complete GPS design information for a rigorous modeling is accessible by elevation and azimuth of the GPS satellites used for the GPS position computation. Additionally, a book keeping of GPS ambiguity terms and their state (fixed or unfixed) is required. The actual vector  $\mathbf{N}$  contains only a counter and a sign to indicate the state. Both information are at hand during the GPS processing. They must be available to estimate coordinate correction in a combined GPS/block adjustment and define the interface between GPS and block adjustment. The GEONAP-K GPS processing package uses undifferenced GPS observable, which makes the handling and processing of the design and ambiguity data very easy. For the use in the block adjustment the design information and the coordinates must be interpolated to the actual event of the photo.

## **5 Geometric Strength and Parameter Separation through Rigorous GPS Model**

GPS gives absolute positions with very high relative accuracy between positions. Therefore the GPS positions can introduce geometric information between individual strips of the complete block. This geometric information is only available, if an adequate model is used. As already pointed out, the very essential geometric information is destroyed by multiple shift and drift parameters in the combined GPS/block adjustment.

The geometric constraints through the rigorous GPS model allows the reduction of ground control points and it is not necessary to have cross strips for the block. The shift & drift approach requires cross strips to overcome the loss of the geometric information inherent in GPS. Even the reduction of side lap is feasible for the rigorous GPS modeling.

The correlation between the interior orientation, namely the focal length and the coordinates of the principal point, datum transformation parameters and shift parameters is very high. Some block adjustment packages even use this high correlation to model systematic GPS errors in the image space instead of the actual object space.

The shift and drift parameters must be distinguished from the transformation parameter between the local coordinate system and the satellite reference system. It is essential to determine the transformation parameters for the block. Shift parameters applied to a complete block and translations of a datum difference cannot mathematically be separated.

From the high correlation of parameters, shift parameters can also not be distinguished from changes of the interior orientation. However, the rigorous GPS approach can separate such error components as the model using the actual satellite constellation and in particular the introduced coordinate corrections due to unresolved ambiguities is different compared to the photogrammetric parameters of the image space.

The correlation between the principal coordinates of the interior orientation with the horizontal component of the GPS positions is getting higher for vertical photographs and hence for a flat terrain. Empirical analysis show, that almost no correlation between these parameters exists in the rigorous modeled GPS/block adjustment. Therefore, the rigorous GPS approach is independent of the topology of the actual terrain.

To get the best geometric condition in the combined GPS/block adjustment, the high relative accuracy of the GPS position has to be maintained. The modeling is independent on the length of the strips and the magnitude and variations of the errors. This is a major aspect of the rigorous GPS modeling approach.

## **6 Rigorous GPS Modeling Using OEEPE Data Set**

Photogrammetric data and GPS data of the IGI photo flight, which is part of phase I, system calibration and direct georeferencing of the OEEPE test, is used. The GPS conditions during the photo flight were in some parts unfavorable, because the weather condition did not allow the flight according to the intended mission planning. The positioning quality of GPS derived from the actual used satellite constellation in the kinematic GPS processing varies from PDOP 1.2 to 4.9.

The GPS processing is based on data from three reference stations (fred, rade, moss) and the kinematic station (figi). The network of reference station gives redundancy, better availability and allows enhanced processing for ambiguity resolution and distance dependent GPS error. The trajectory has been computed in the ETRF89 datum defined by the coordinates of station fred. The coordinates of the GPS antenna were transferred into the UTM projection on the WGS84 ellipsoid and interpolated for the recorded event times of the photos. The uncertainty of the GPS position at the stage of the combined adjustment consists of several different parts. These are the GPS processing, the time synchronization of events and the interpolation. While the accuracy of the processing is in the order of 0.05–0.10 m, the accuracy of the events is only 0.5 ms. From the velocity of the airplane of ca. 100 m/s during the flight, an uncertainty of up to 5 cm results from the time synchronization. The interpolation error is expected to be small due to the overall recording interval of 2 Hz for the GPS data. The eccentricity of the GPS antenna is applied in the block adjustment, because the additional orientation information from AT or IMU can be applied. The eccentricity vector is generally assumed to be precisely known. The datum transformation can approximately done in a first step before the combined adjustment. The local datum differences are best estimated in the combined GPS (block adjustment itself having generally additional data).

Figure 3 shows the available satellites from the original recorded RINEX data on three reference stations and the kinematic station, as well as the actual used satellites of the kinematic station.

### SV Availability RINEX/Processing

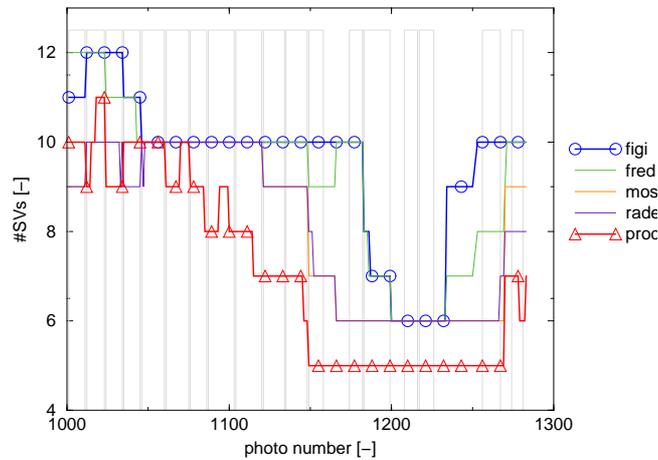


Figure 3: GPS satellite constellation and strips

Absolute results from the combined processing with GEONAP-K and BINGO-F cannot be presented from the present data, because independent check points are not published in phase I, calibration photo flights and georeferencing. Therefore, a kind of extended data set is used in one adjustment, which consists of available data from both companies participating with a GPS/IMU system in the OEEPE test. According to phase I, the exterior orientation may be estimated from these calibration data sets.

The analysis of the complete IGI block applying the rigorous GPS model (CPAS) show in some parts of the block very large misclosures. There are obviously systematic effects in the residuals of the projection center as shown in figure 5. The effects cannot be eliminated with a complete self-calibration of the camera and additional parameters. Therefore some detailed analysis of the GEONAP-K processing and the estimated coordinates were executed, which showed no errors or causes from the GPS data or processing. Investigations concerning any problems in the determination of image coordinates had also no result.

The residuals of the IGI block apparently originate from the coordinates of the principal point of the camera. This became obviously after numerous analysis and investigation of the photogrammetric data, also together with other researchers (Cramer 2001). The capability of the rigorous GPS model approach to separate between individual parameters of interest is used to determine corrections for the principal point. The systematic effects of figure 4 disappear completely after applying different camera parameters for parts of the block (see table 4). There are major differences especially in the y-component of the principal point, which are high significant considering the standard deviation. Afterwards, the complete block does not show any significant residuals (figure 5).

For verification of this findings, all four individual block provided in phase I (calibration flight 1:5000, calibration flight 1:10000, block and strip) of both companies are processed as a free network with self-calibration of the principal point. Table 3 shows the variations of the principal point for several different adjustment strategies in the block adjustment. The standard deviation indicates, that the corrections of the principal points are not significant. However, there is a general trend, which agrees with the results of table 2.

<i>Block Name</i>	<i>xH'</i>	<i>yH'</i>	<i>S xH'</i>	<i>S yH'</i>	<i>Remarks</i>
C1 – CPAS adjustment with 3 camera numbers	+3.9	-12.8	+1.2	+1.2	Cam1
	+11.0	+12.7	+2.3	+2.4	Cam2
	+10.2	+7.9	+3.4	+3.4	Cam3: Cass2

Table 2: Estimated principal point  $xH'$ ,  $yH'$  and standard deviation from combined GPS/block adjustment with ground control points

<i>Block Name</i>	<i>xH'</i>	<i>yH'</i>	<i>S xH'</i>	<i>S yH'</i>	<i>Remarks</i>
C1 – part of block, divided by used cassette	-5.9	-0.8	+10.5	+11.4	Cam1: Cass1
	+0.1	+13.3	+22.4	+23.9	Cam2: Cass2
C1 – complete block, divided by used cassette	-1.0	-8.9	+5.0	+5.1	Cam1: Cass1: 202 photos
	-11.2	+7.7	+19.2	+21.1	Cam2: Cass2: 15 photos
C1 – all photos	-0.1	-6.6	+4.7	+4.8	Cass1: 202 photos Cass2: 15 photos
C1 – calibration 1:5000/1:10000	+1.6	-10.3	+5.5	+5.5	Cass1: all photos
C1 – Block+Strip	-2.1	+5.1	+8.6	+9.2	Cass1: 54 photos Cass2: 15 photos

Table 3: Estimated principal point  $xH'$ ,  $yH'$  and standard deviation from free network bundle block adjustment, IGI data, company 1

<i>Block Name</i>	<i>xH'</i>	<i>yH'</i>	<i>S xH'</i>	<i>S yH'</i>	<i>Remarks</i>
C2 – all photos	+11.3	+18.8	+4.5	+4.7	
C2 – Calibration 1:5000/1:10000	+14.0	+20.9	+6.1	+6.1	
C2 – Block+Strip	+19.7	+22.9	+7.4	+7.8	

Table 4: Estimated principal point  $xH'$ ,  $yH'$  and standard deviation from free network bundle block adjustment, Applanix data, company 2

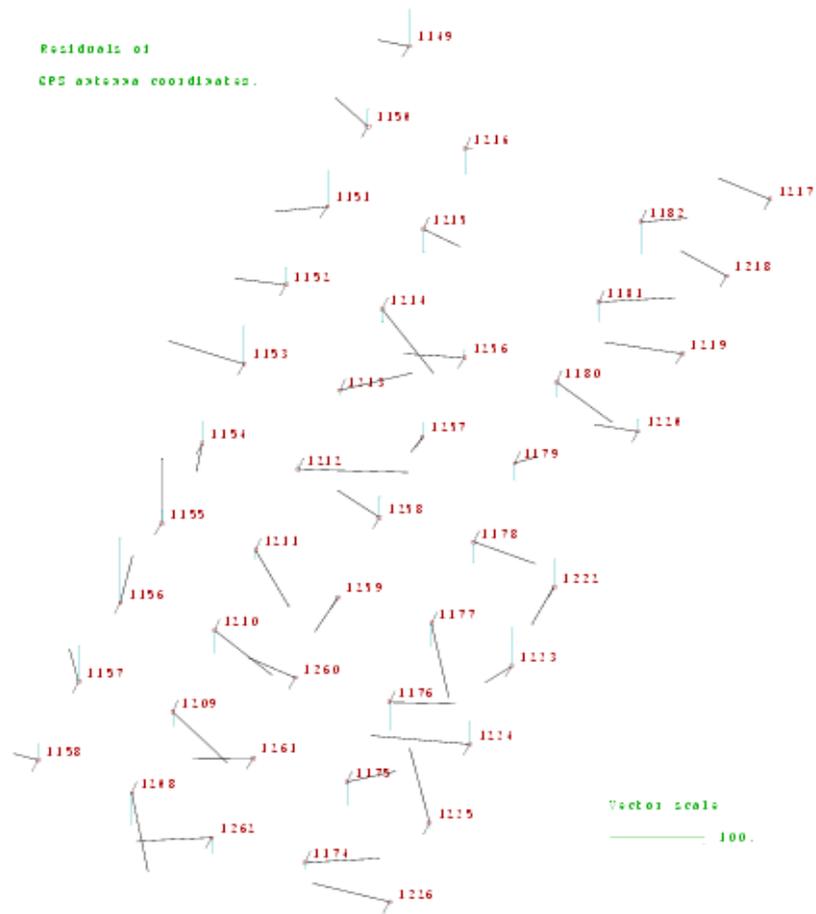


Figure 4: Section from the complete IGI photo flight with large systematic effects (horizontal and vertical residuals, small vectors indicating flight direction), units are mm



Table 4 shows the variations of the principal point for several different adjustment strategies in the free network block adjustment of company 2. The principal point is significantly determined and is verified within the different block configurations.

Generally, comparable misclosures are present in the processing of the complete Applanix block using the provided GPS coordinates for the projection centers (not shown). However, the residuals vanish after introducing one set of unknowns for the principal point of the camera in the adjustment. For the size of the principal point corrections see table 4. The processing and comparison with the rigorous GPS model was not possible, because the GPS raw data were not available.

While the principal point of the IGI block shows again large differences (table 3), the principal point of the Applanix block is stable (table 4). In the free network processing only the photogrammetric data is used. Hence, the GPS/IMU processing results of IGI and Applanix do not have any influence on the results.

The selection of the partial blocks is somehow arbitrary, leading to the not solvable question of the adequate choice for the determination of the different locations of the principal point. Some processing results even indicate, that for some part of the block the differences in the principal point coordinates are much higher.

## **7 Benefits of Rigorous Modeling of GPS**

The benefits of coordinate corrections from the rigorous GPS modeling in the combined GPS/block adjustment have been discussed in the previous chapters. The restriction in the available data made absolute results using independent control points not yet possible. However, from the discussed theory, analysis and from our empirical experiences, the following list summarizes the major advantages of a rigorous GPS modeling in the combined GPS/block adjustment:

- correct modeling of all GPS errors
- independent of strips
- considers the actual GPS model
- considers drift and GPS constellation changes
- reduced number of unknowns
- relative accuracy of GPS coordinates is maintained
- no crossing strips required
- enables separation of systematic GPS errors from i.e. datum parameters, additional parameters of interior orientation
- reduction of side lap possible

## **8 Aspects for Integration of AT/GPS/IMU**

The current attempts in aerial triangulation are to integrate GPS and IMU data for georeferencing. The interest is again to reduce the costs of a photogrammetric survey by substituting photogrammetric data by IMU data. Our experiences with the rigorous GPS modeling show, that also a simultaneous, combined adjustment of GPS/IMU/AT can benefit from a closed approach. It might be necessary to develop special configurations of ground control points and special procedures for the time sequence of flying strips. One particular calibration flight is considered as

not sufficient to model remaining systematic GPS position effects adequately. It might work for certain accuracy requirements, but technical development and adoption of techniques for other applications and accuracy specifications proceed, which makes further investigations useful.

Nevertheless, the correct GPS modeling of remaining error requires the knowledge of the processing involved in all processing steps. The integration of IMU and GPS data must be known at least in some details to decide upon the model to be used in the combined adjustment. On the one hand the IMU data can be used solely as a sensor of orientation in addition to GPS for positioning, on the other hand the IMU data can be integrated for positioning and coupled with GPS data for a combined trajectory. In the latter case, the rigorous model as well as simple shift & drift approximation for remaining systematic GPS errors might fail without the knowledge of the processing.

The accuracy of orientation data from an IMU is generally not sufficient to significantly constrain the external orientation of AT. However, the intention of the use of IMU data is the transfer of exterior orientation with a reduction of ground control points and photogrammetric data. It is essential for this task, that the parameter of interior orientation can be separated from the exterior orientation. The calibration of the camera's principal point must be accurate to 20  $\mu\text{m}$ , because in dependency of the actual photo scale significant errors are possible for the coordinates in object space. Again, there exists a high correlation between IMU data and the principal point. The separation of these error components is only practicable with the rigorous GPS model in the combined adjustment.

## **9 Conclusion**

The rigorous GPS modeling in the combined GPS/block adjustment has been explained. The advantages and benefits of the approach and comparisons with the shift & drift approach have been discussed. The rigorous GPS model in the combined GPS/block adjustment uses the actual GPS satellite geometry and keeps the geometric relationship between individual strips and the complete block. The strengthening of geometry becomes obvious as crossing flight strips can be completely dropped, even for blocks with few control points. The rigorous GPS approach allows to estimate GPS position corrections for a complete block using strictly the functional GPS model. Hence, the correlation with other parameters of interest is significantly reduced, which allows to account for individual error components of the block adjustment.

The use of IMU data in the combined block adjustment is encouraging, although no actual IMU data has been used in this paper, benefits for a closed adjustment of GPS/IMU/AT from the rigorous GPS modeling are expected. Additional investigation and analysis is required in this respect.

At the time of writing, the data of the OEEPE test is restricted. There are no independent checks for absolute comparisons available or other useful comparisons of the rigorous GPS modeling using GEONAP-K/BINGO-F were possible. The check points will be made available in a later phase of the OEEPE test, and will then be used to completed and report the investigations.

After numerous investigation and analysis of the photogrammetric part of the OEEPE test data, it must be assumed, that differences in performance and accuracy of the two data set within the OEEPE test might be caused by the provided photogrammetric data and not necessarily by differences of the GPS/IMU systems of the companies IGI and Applanix. There are a lot of steps involved from picture taking to image coordinate determination, which in general are all capable to introduce the detected effect. However, the principal point is an essential part of the

photogrammetric coordinate determination of the OEEPE test, which even can make results indeterminate as long as a varying principal point location is actually considered possible. A likely cause has not been brought up here and is left for discussion within the actual OEEPE test.

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