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# Precise Kinematic GPS Processing and Rigorous Modeling of GPS in a Photogrammetric Block Adjustment

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# Motivation and Development

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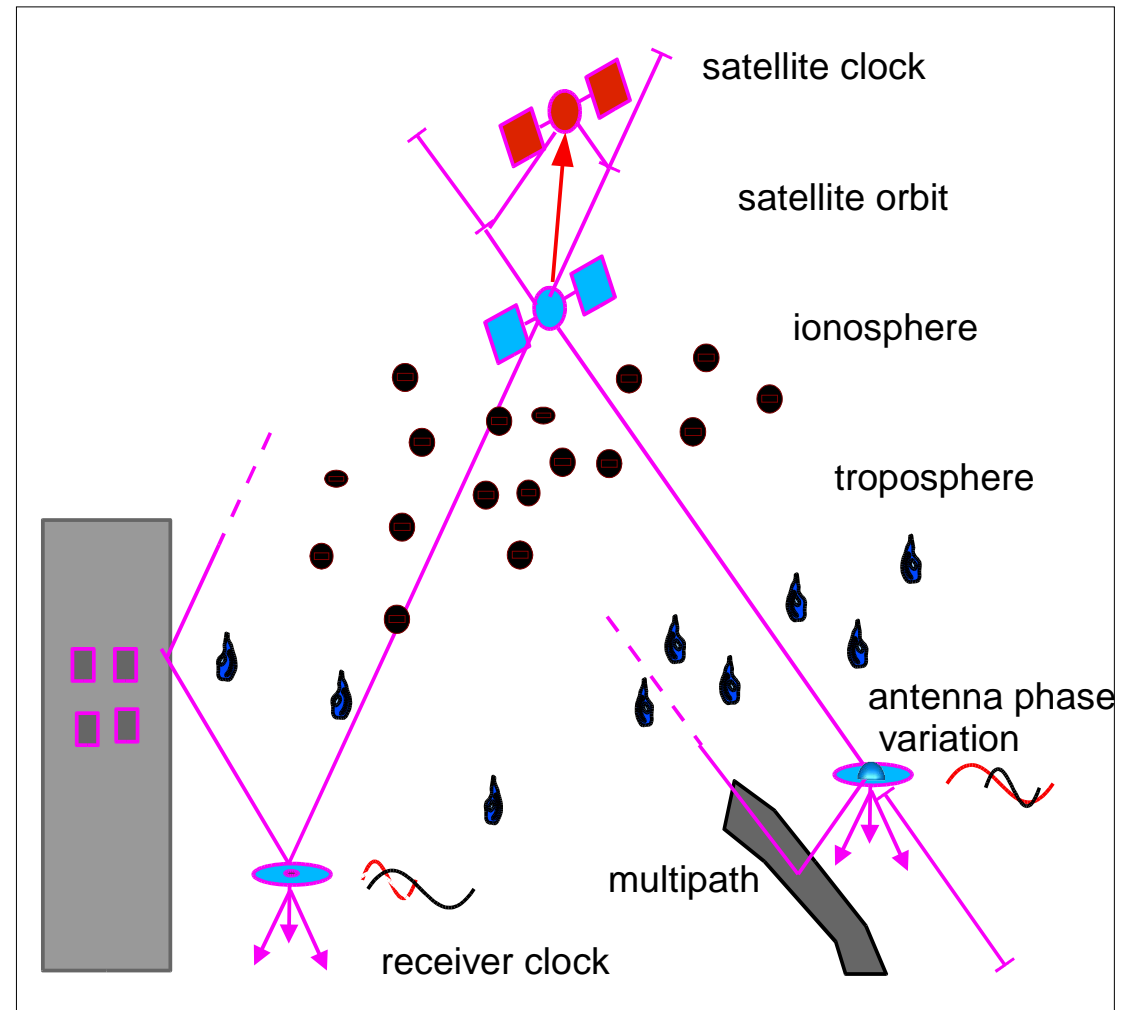


- precise kinematic GPS processing
- challenging GPS task
- combination of GPS and photogrammetric applications
- handling of possible GPS coordinate errors
- generally approximation without reflecting GPS model applied
- research project at University of Hannover in 1995-1996
  - rigorous GPS Model developed for combined GPS/block adjustment
- GEONAP-K GPS- / BINGO block adjustment
- operationally applied since 1996

# GPS Error Sources



- **station dependent error**
  - antenna (center) phase variation (PCV)
  - multipath
- **distance dependent error**
  - ionosphere
  - troposphere
  - satellite orbit
- kinematic GPS: **additional systematic coordinate effects**
  - constellation changes
  - approximate or false ambiguity resolution



# Magnitude of GPS Error Sources



error source	absolute influence	relative influence
satellite orbit	2 ... 50 m	0.1 ... 2 ppm
clock	2 ... 100 m	0.0 ppm
ionosphere	0.5 ... >100 m	1 ... 50 ppm
troposphere	0.01 ... 0.5 m	0 ... 3 ppm
multipath code	m	m
multipath phase	mm ... cm	mm ... cm
antenna	mm ... cm	mm ... cm

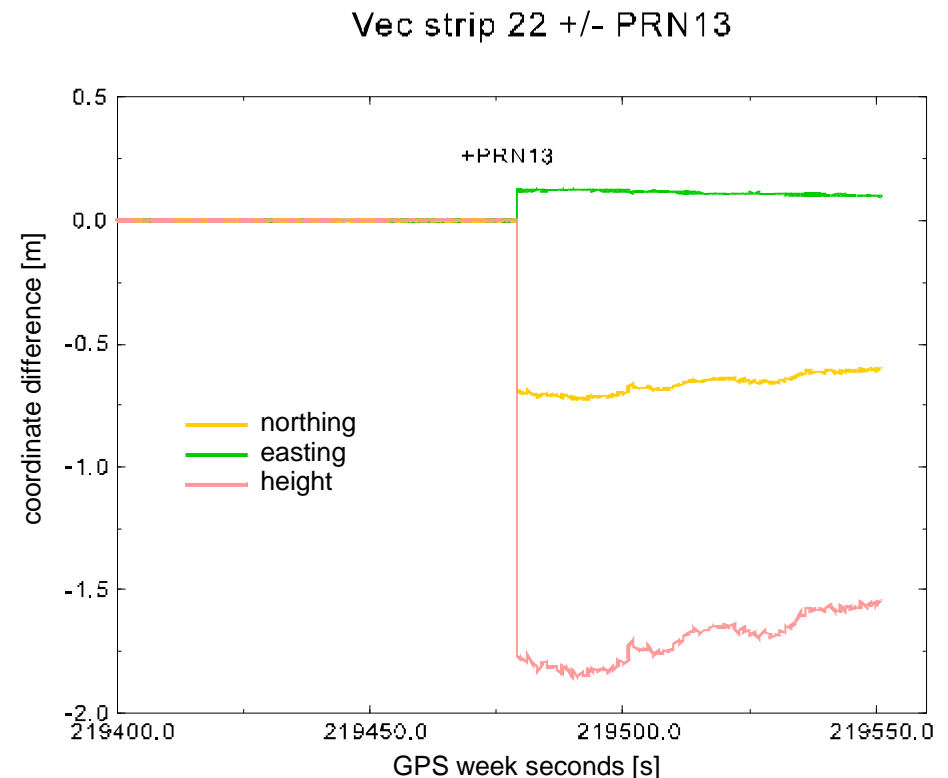
 high spatial correlation

 local (calibration)

# Systematic GPS Coordinate Errors



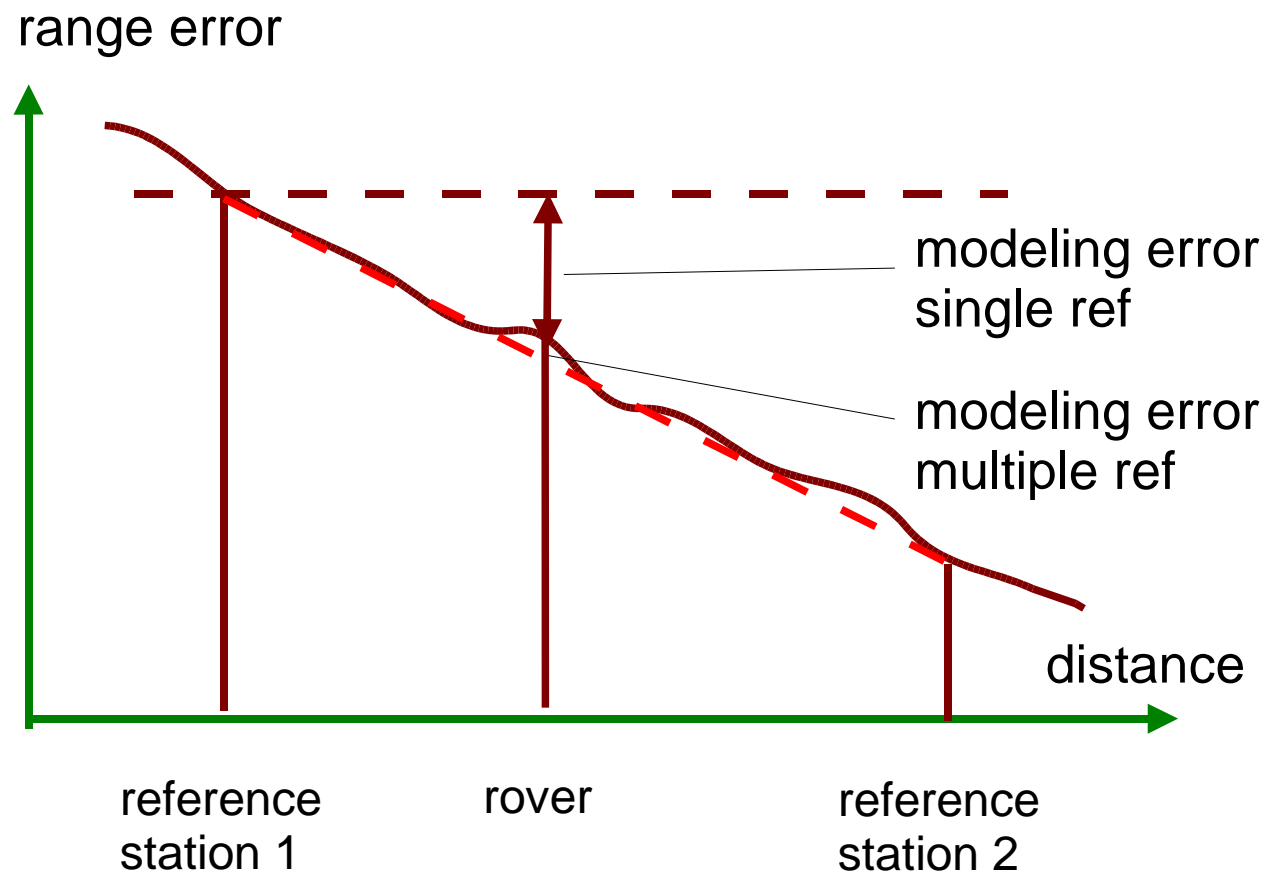
- systematic GPS coordinate errors may be present due to
  - false ambiguity fixing (time dependent effects)
  - changes in the satellite constellation (discontinuity)
  - GPS error sources
- magnitude of errors depends on
  - geometric GPS conditions (DOP values)
  - known systematic GPS effects
- modeling of these errors attempted in combined adjustment of GPS and aerial triangulation



# Multiple Reference Stations



- spatial correlation of GPS errors
- distance dependency
- **advantage to use multiple reference stations**
- more redundancy
- allows better modeling of GPS errors
- improved coordinates



# Geo++<sup>®</sup> GEONAP

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- GEONAP - *Geodetic Navstar Positioning*
  - **multi-signal, multi-station**, multi-session adjustment (rigorous adjustment of different signals and multiple kinematic and/or static stations)
  - **undifferenced observable** with **complete variance-covariance estimation**
  - consideration of all major error components
- development and maintenance by Geo++<sup>®</sup> - since 1990
- advanced GPS software
  - for static and kinematic applications
  - for small, large and regional applications
  - different accuracy levels from mm ... m



# Considerations for Kinematic GPS



<i><b>Kinematic GPS Processing</b></i>	<i><b>local reference station</b></i>	<i><b>remote reference station</b></i>	<i><b>reference station network</b></i>
ambiguity resolution	possible	difficult	possible
distance dependent errors:			
• ionosphere	ignore, <b>estimate</b> , eliminate	ignore, <b>estimate</b> , eliminate	<b>estimate</b> , eliminate
• troposphere	<b>model</b> , estimate	<b>model</b> , estimate	<b>model</b> , estimate
• orbit	BE, <b>PE</b>	BE, model, <b>PE</b>	BE, model, <b>PE</b>
remaining systematic effects:			
• shift, drift of coordinates	approximate, <b>rigorous model</b>	approximate, <b>rigorous model</b>	approximate, <b>rigorous model</b>
• antenna PCV	<b>correct</b>	<b>correct</b>	<b>correct</b>
costs	high	low	low

- all major effects can be corrected or modeled
- consider costs for choice on reference station(s)

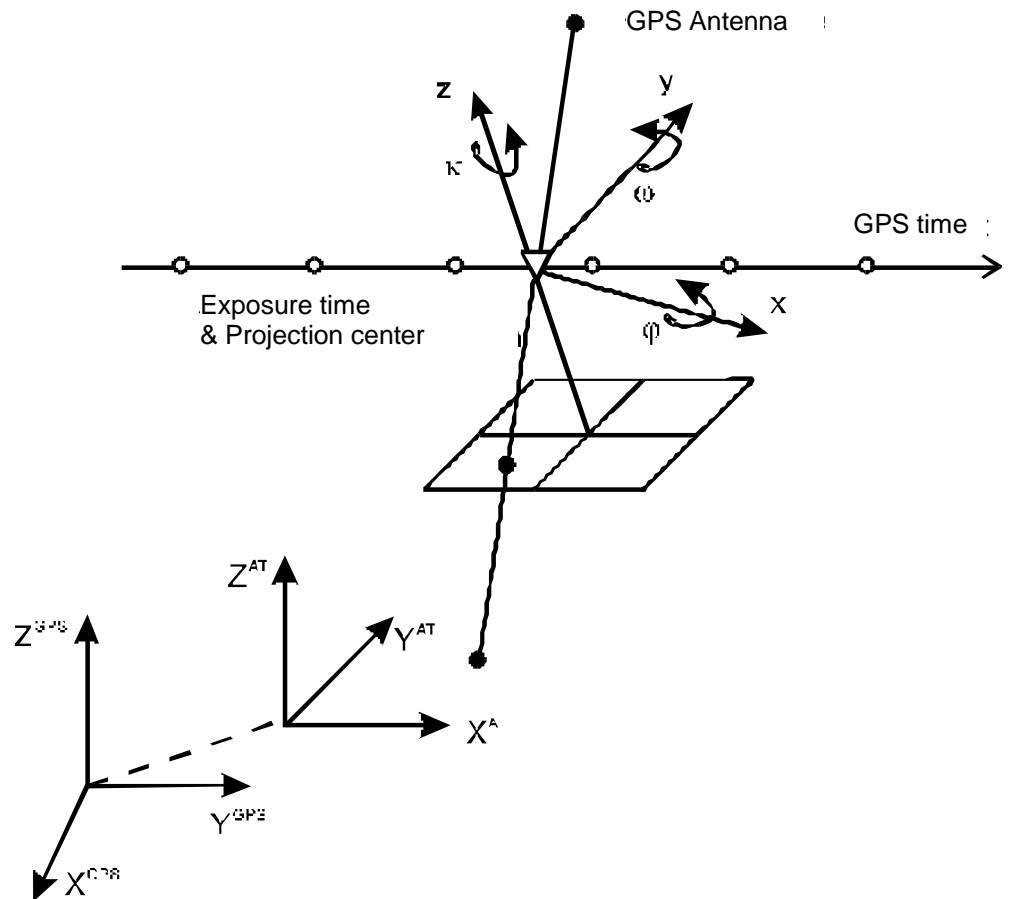
**■** recommended, but depends very much on application and data

BE broadcast ephemeris  
PE precise ephemeris  
PCV phase (center) variations

# Relating GPS and Aerial Triangulation



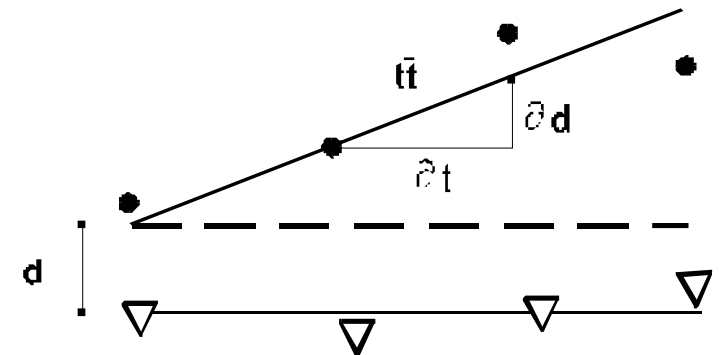
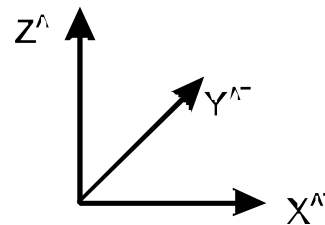
- pre-requisite: unchanged or known conditions of antenna/camera for the complete photo-flight
  - **identical reference point** by orientated vector antenna/camera
  - **identical reference time** by interpolation of GPS coordinates or synchronization of GPS and camera
  - **identical geodetic datum** by datum transformation and/or adjustment



# Shift- and Drift- Approximation in GPS/Block Adjustment



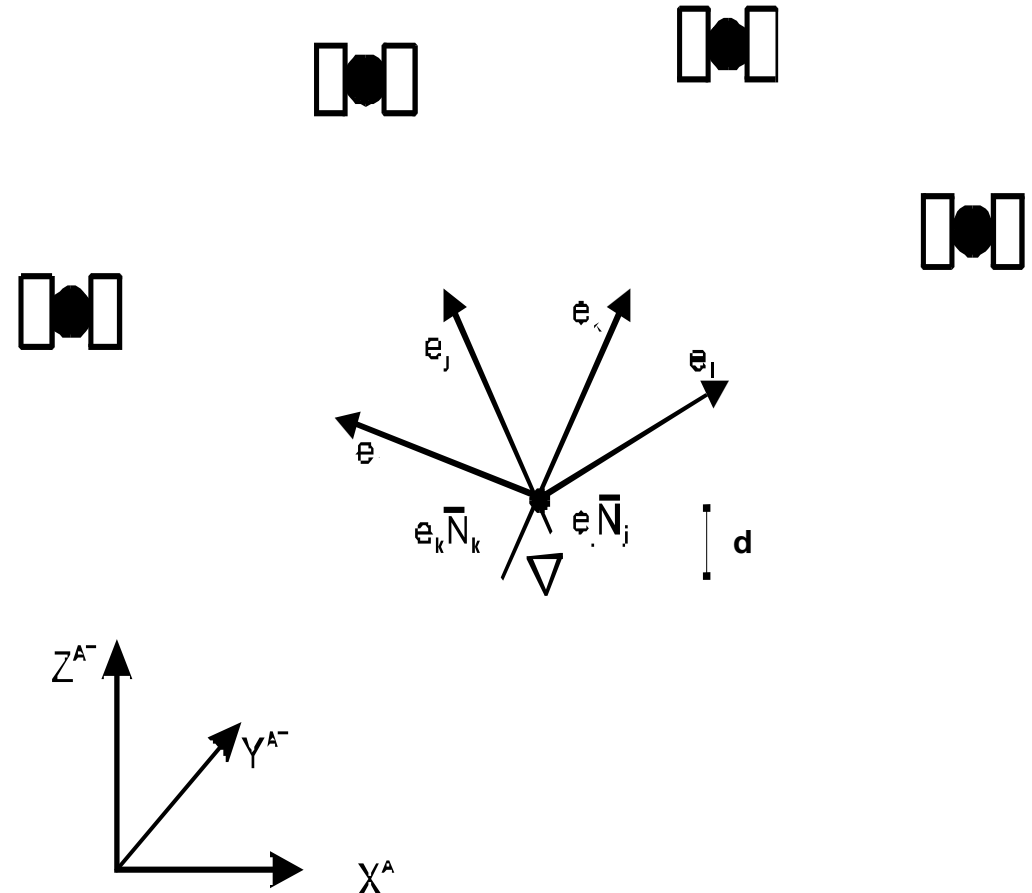
- projection center • GPS and  $\nabla$  AT
- functioning of the shift- and drift-parameter
  - translation and time/strip dependent corrections
    - generally for every strip
    - individual for coordinate components
    - no relationship between strips by GPS
    - considers no changes of satellite constellation
    - approximation of actual GPS model



# Rigorous GPS Model in GPS/Block Adjustment



- projection center • GPS and  $\nabla$  AT
- functioning of rigorous GPS modeling
  - unreliable ambiguities / constellation changes
    - considers actual GPS satellite constellation
    - estimates range/position correction
    - keeps geometric GPS relationship
    - reduces correlation with other parameter



# Comparison of the Mathematical Models



- rigorous GPS model

$$X_{P_i}^{AT} = X_{A_i}^{GPS} + dX_D + (QA^T P)_i * \bar{N}_i + R_i(\phi \omega \kappa) * dX_A$$

- shift and drift approach

$$X_{P_i}^{AT} = X_{A_i}^{GPS} + dX_D + (dSP_i) + R_i(\phi \omega \kappa) * dX_A$$

$X_{P_i}^{AT}$	coordinates of projection center
$X_{A_i}^{GPS}$	(interpolated) coordinates GPS antenna
$dX_D$	datum transformation
$QA^T P_i$	GPS design information
$\bar{N}_i$	ambiguity/range term
$R_i(\phi \omega \kappa)$	rotation matrix from block adjustment/IMU
$dX_A$	vector GPS antenna/projection center
$dSP_i$	shift & drift parameter term
$i$	exposure i

# Interface between GPS and Block Adjustment



- complete design-information accessible by elevation  $e$  and azimuth  $a$  of the GPS satellites

$$A_j = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_k \end{bmatrix} \quad a_k^T = \begin{bmatrix} e_x \\ e_y \\ e_z \\ c_0 dt \end{bmatrix} = \begin{bmatrix} -\cos e \cos \alpha \\ -\cos e \sin \alpha \\ -\sin e \\ 1 \end{bmatrix}$$

- book keeping of GPS ambiguities  $N$  (wavelength  $\lambda$ )

$$\bar{N}_i = \begin{bmatrix} \bar{N}_1 \\ \bar{N}_2 \\ \dots \\ \bar{N}_k \end{bmatrix} = \lambda \cdot \begin{bmatrix} N_1 \\ N_2 \\ \dots \\ N_k \end{bmatrix} \quad N_k = \begin{cases} 0 & \text{reliable fixed} \\ 1 & \text{not reliable fixed} \end{cases}$$

- estimation of coordinate correction in combined adjustment

$$dX_i = (QA^T P_i) * \bar{N}_i$$

# Benefits of Rigorous Modeling of GPS

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- correct modeling of all GPS errors
- independent of strips/block
- considers the actual GPS model
- considers time dependent effects and GPS constellation changes
- reduced number of unknowns in combined adjustment
- relative accuracy of GPS coordinates is maintained (for all strips and complete block)
- no crossing strips required
- separation of systematic GPS errors from
  - e.g. datum parameters
  - additional parameters e.g. interior orientation
- reduction of ground control points and side lap possible
- cost reduction feasible

# GEONAP/BINGO

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- GPS processing for **photogrammetric application**
  - simultaneous adjustment of several kinematic rovers and reference station data possible
- **sophisticated feature**: subsequent processing with  
  
GEONAP-K package for GPS data and  
BINGO\* for combined adjustment
- only operationally applied **rigorous GPS modeling in block adjustment**
- also termed CPAS (Combined Phase Ambiguity Solution)

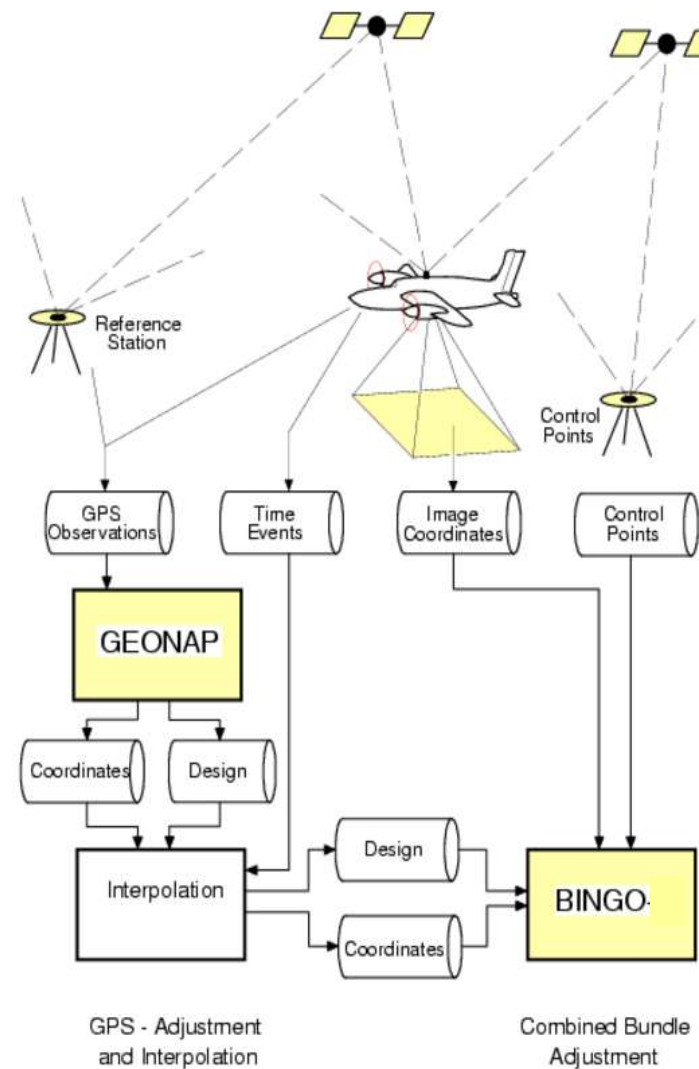
\* BINGO from GIP  
Geoinformation & Photogrammetric Engineering  
D-73430 Aalen  
[www.gip-aalen.de](http://www.gip-aalen.de)



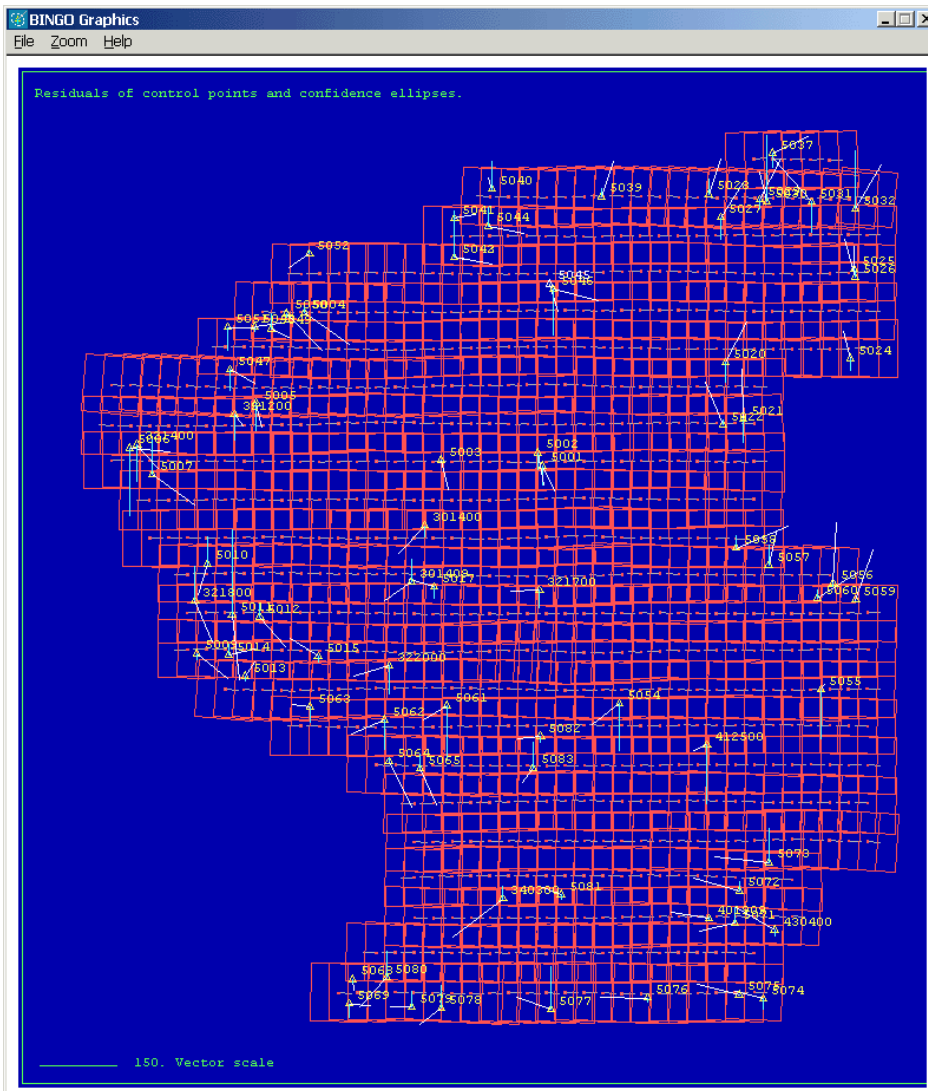
# Data Flow GEONAP/BINGO



- standardized precise kinematic GPS processing
- provides trajectory of kinematic GPS antenna
- provides design information and ambiguity status for every event
- not necessary to solve all ambiguities
- coordinate transformation/interpolation
- complete information at hand to rigorously model GPS in block adjustment
- influence of “unfixed ambiguities” estimated



# Example of Rigorous Approach Fehrbellin

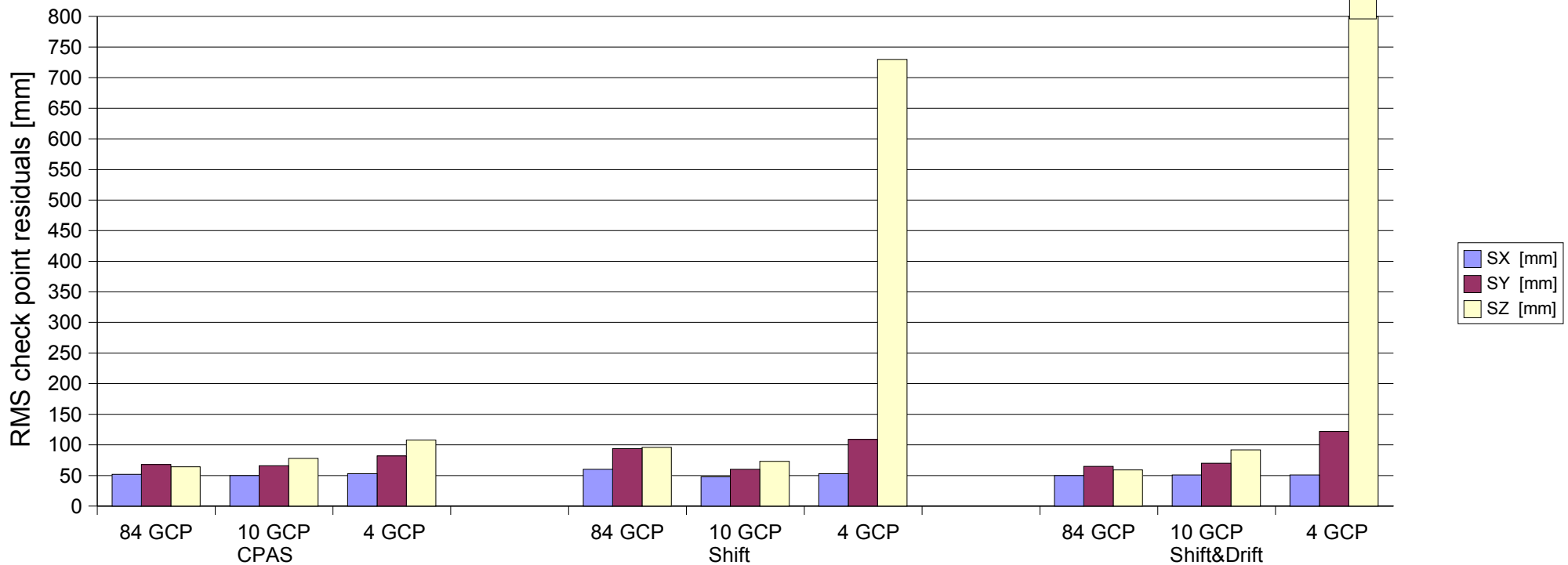


- dataset Fehrbellin, Germany, 1999
- scale 1:3500
- 84 ground control points
  - not very precise (~5 cm)
- GPS with severe signal interruptions
- following analysis using
  - no cross-strips
  - reduced number of ground control points (84 ... 10 ... 4)
  - comparing
    - rigorous approach (CPAS)
    - shift-, shift & drift approach

# Example of Rigorous Approach Fehrbellin



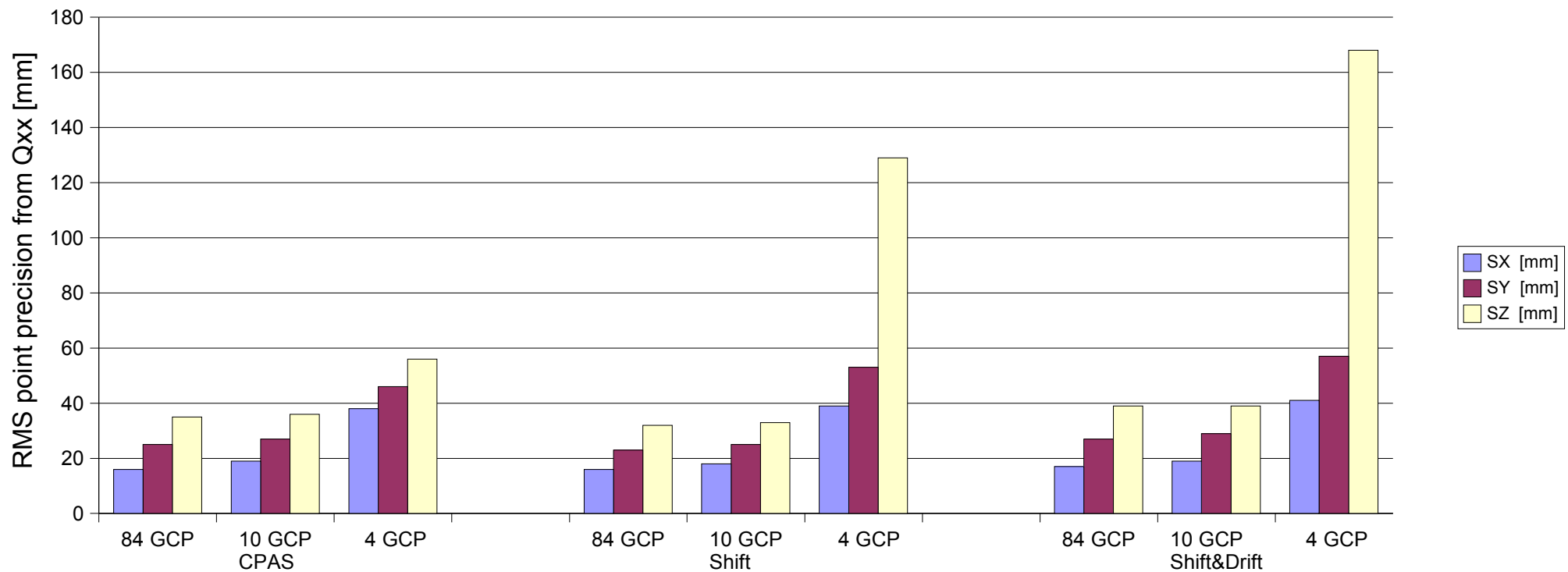
- RMS check point residuals for
  - different number of ground control point
  - different GPS modeling



# Example of Rigorous Approach Fehrbellin



- RMS point precision from combined GPS/block adjustment
  - small differences due to quality of ground control point



# Example of Rigorous Approach Fehrbellin



- dataset Fehrbellin
- numerical values as derived from rigorous modeling of GPS in combined GPS block adjustment  
GEONAP/BINGO

	CPAS			Shift			Shift&Drift		
RMS check point residuals [mm]									
	SX	SY	SZ	SX	SY	SZ	SX	SY	SZ
84 CGP	52	50	53	60	48	53	50	51	51
10 CGP	68	66	82	94	60	109	65	70	122
4 CGP	64	78	108	96	73	730	59	92	1657

RMS point precision from Qxx [mm]									
	SX	SY	SZ	SX	SY	SZ	SX	SY	SZ
84 CGP	16	19	38	16	18	39	17	19	41
10 CGP	25	27	46	23	25	53	27	29	57
4 CGP	35	36	56	32	33	129	39	39	168

# Summary

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- precise kinematic GPS processing revisited
  - all major error components can be corrected or modeled
  - advantages of multiple reference stations
- rigorous GPS modeling in combined GPS/block adjustment revisited
  - uses the actual GPS satellite geometry
  - advantages and benefits of approach
    - keeps geometric GPS relationship / strengthening of geometry in block adjustment
    - correct functional GPS model / reduced correlation with other parameter
    - operational experiences underline advantages: reduction of ground control points, no cross-strips
- operational procedure with GEONAP-K and BINGO since 1996



for publications on the presented topic refer also to

[www.geopp.com](http://www.geopp.com)

or directly to

[http://www.geopp.com/publications/english/lit\\_e.htm](http://www.geopp.com/publications/english/lit_e.htm)